

Department of Mathematics
University of North Carolina at Chapel Hill
Chapel Hill, North Carolina

The Alfred Brauer Lectures 2016

Stan Osher

University of California at Los Angeles

**LECTURE 1: What mathematical algorithms can do for the real
(and even fake) world**

Wednesday, April 20, 2016 from 3:30 – 4:30*
Phillips Hall, Room 215

LECTURE 2: What Sparsity and l1 Optimization Can Do For You

Thursday, April 21, 2016 from 4:00 – 5:00
Phillips Hall, Room 332

**LECTURE 3: Overcoming the curse of dimensionality for certain
Hamilton-Jacobi (HJ) equations arising in control theory and elsewhere**

Friday, April 22, 2016 from 4:00 – 5:00
Phillips Hall, Room 332

*There will be a reception in the Mathematics Faculty/Student Lounge on the third floor of Phillips Hall, Room 330, 4:45—6:00 pm, on Wednesday, April 20, 2016. Refreshments will be available there at 3:30 before the second and third lectures.

See website for complete abstract: <http://math.unc.edu/seminars-and-colloquia/brauer>

Stan Osher

University of California at Los Angeles

Brauer Lectures

LECTURE 1: What mathematical algorithms can do for the real (and even fake) world

Abstract: I will give a very personal overview of the evolution of mainstream applied mathematics from the early 60's onwards. This era started pre computer with mostly analytic techniques, followed by linear stability analysis for finite difference approximations, to shock waves, to image processing, to the motion of fronts and interfaces, to compressive sensing and the associated optimization challenges, to the use of sparsity in Schrodinger's equation and other PDE's, to overcoming the curse of dimensionality in parts of control theory and in solving the associated high dimensional Hamilton-Jacobi equations.

LECTURE 2: What Sparsity and l1 Optimization Can Do For You

Abstract: Sparsity and compressive sensing have had a tremendous impact in science, technology, medicine, imaging, machine learning and now, in solving multiscale problems in applied partial differential equations, developing sparse bases for Elliptic eigenspaces and connections with viscosity solutions to Hamilton-Jacobi equations. l1 and related optimization solvers are a key tool in this area. The special nature of this functional allows for very fast solvers: l1 actually forgives and forgets errors in Bregman iterative methods.

I will describe simple, fast algorithms and new applications ranging from image processing, machine learning to sparse dynamics for PDE.

LECTURE 3: Overcoming the curse of dimensionality for certain Hamilton-Jacobi (HJ) equations arising in control theory and elsewhere

Abstract: It is well known that certain HJ PDE's play an important role in analyzing continuous dynamic games and control theory problems. The cost of standard algorithms, and, in fact all PDE grid based approximations is exponential in the space dimension and time, with huge memory requirements.

Here we propose and test methods for solving a large class of HJ PDE relevant to optimal control without the use of grids or numerical approximations. Rather we use the classical Hopf formulas for solving initial value problems for HJ PDE. We have noticed that if the Hamiltonian is convex and positively homogeneous of degree one that very fast methods (related to those used in compressed sensing) exist to solve the resulting optimization problem. We seem to obtain methods which are polynomial in dimension. We can evaluate the solution in very high dimensions in between 10^{-4} and 10^{-8} seconds per evaluation on a laptop. The method requires very limited memory and is almost perfectly parallelizable.

In addition, as a step often needed in this procedure, we have developed a new and equally fast and efficient method to find, in very high dimensions, the projection of a point exterior to a compact set A onto A . We can also compute the distance to such sets much faster than fast marching or fast sweeping algorithms.

The term "curse of dimensionality" was coined by Richard Bellman in 1957 when he did his pioneering work on dynamic optimization.